Exploring driven dissipative Spin 1/2 dynamics within open quantum system.

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Aim of this talk

- Open quantum system and Quantum Master Equation.
- Application on dissipative Spin 1/2 system.

Motivation

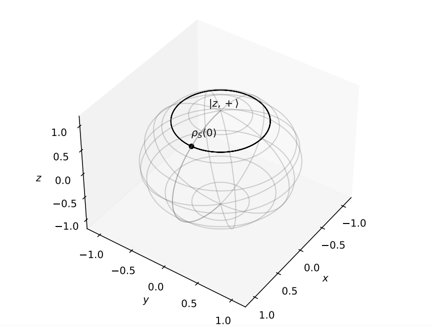
Simple spin 1/2 system

- Spin 1/2 system in constant magnetic field $\vec{B} = (0, 0, B)$
- The system Hamiltonian: $H_S^\circ = \frac{g\mu_B B}{2}\sigma_z = \omega_0 I_z$.
- The system state: $|\psi\rangle$ and $\rho_S = |\psi\rangle\langle\psi|$.
- Von-Neumann equation:

$$
\begin{aligned}
&\dot{\rho}_S(t) = -i[H_S^\circ, \rho_S(t)] = -i\omega_0[I_z, \rho_S(t)] \\
\Leftrightarrow &\, |\dot{\psi}\rangle = -i\omega_0 I_z |\psi\rangle
\end{aligned}
$$

- Start with state $|\psi_0\rangle = \frac{3}{5}|x,+\rangle + \frac{4}{5}|z,+\rangle$
- Evolve by Von-Neumann equation.

Simple spin 1/2 system



- Precession on the surface of Bloch sphere (pure state).
- Unitary evolution, NO dissipation.

Driven spin 1/2 system

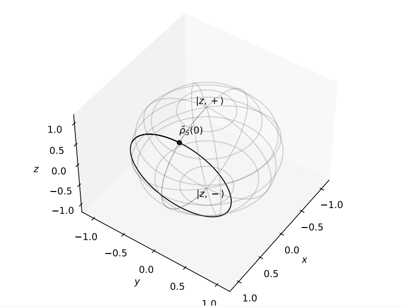
- Add periodic drive as $\vec{B}_{\text{drive}} = (2B_1\cos(\omega_0 t), 0, 0)$ where $B_1 \ll B$
- Add Hamiltonian $H_D(t) = 2\omega_1\cos(\omega_0 t)I_x$ where $\omega_1 \ll \omega_0$
- $H = H_S^\circ + H_D(t)$
- **Interaction picture:** Rotating frame $\rightarrow \tilde{O}(t) = e^{iH_S^\circ t}O(t)e^{-iH_S^\circ t}$
    - Hailtonian:

$$\tilde{H}_D(t) = e^{i\omega_0 t I_z}\, H_D(t)\, e^{-i\omega_0 t I_z}$$

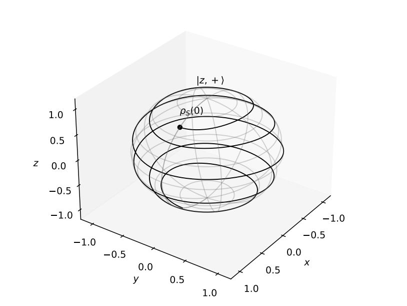
    - State: $\tilde{\rho}_S(t) = e^{i\omega_0 t I_z}\,\rho_S(t)\,e^{-i\omega_0 t I_z}$
    - Von-Neumann equation:

$$\dot{\tilde{\rho}}_S(t) = -i[\tilde{H}_D(t), \tilde{\rho}_S(t)]$$

Driven spin 1/2 system



(a) Rotating frame



(b) Lab frame

- Still on the surface of Bloch sphere (pure state).
- Unitary evolution, NO dissipation.
- **Drawback**: Too ideal to be real.
    - Real system in contact with environment.
    - Have dissipation, decoherence, relaxation.

Formulation of Open Quantum System

Open quantum system

- Deals with system interacting with environment.
- Total state: $\rho_T = \rho_S \otimes \rho_R$
- Hamiltonian of the system: H_S°
- Hamiltonian of the drive: H_D
- Hamiltonian of environment: H_B° (non-stochastic)
- Hamiltonian of environment: H_{st} (stochastic)
- Hamiltonian of the interaction: H_{SR}
- Total Hamiltonian:

$$
\begin{aligned}
H &= H_S^\circ \otimes \mathbb{1} + \mathbb{1} \otimes H_R^\circ + H_{SR} + H_D(t) \otimes \mathbb{1} + \mathbb{1} \otimes H_{st}(t) \\
&\equiv H_S^\circ + H_R^\circ + H_{SR} + H_D(t) + H_{st}(t) \\
&= H_S^\circ + H_R^\circ + V(t)
\end{aligned}
$$

‘Fluctuation Regulated’ Quantum Master Equation

- **Interaction picture**: $\tilde{O}(t) = e^{i(H_S^\circ + H_R^\circ)t} O(t) e^{-i(H_S^\circ + H_R^\circ)t}$
  - Von-Neumann equation:

$$\dot{\tilde{\rho}}_T(t) = -i[\tilde{V}(t), \tilde{\rho}_T(t)]$$
$$\tilde{\rho}_T(t+\delta t) = \tilde{\rho}_T(t) - i\int_t^{t+\delta t} dt_1 [\tilde{V}(t_1), \tilde{\rho}_T(t_1)]$$

- Interested in only system evolution.

$$\tilde{\rho}_S(t) = \mathrm{Tr}_R\left(\tilde{\rho}_T(t)\right)$$

- $\tilde{\rho}_S(t+\delta t) = \tilde{\rho}_S(t) - i\int_t^{t+\delta t} dt_1\, \mathrm{Tr}_R[\tilde{V}(t_1), \tilde{\rho}_T(t_1)]$

'Fluctuation Regulated' Quantum Master Equation

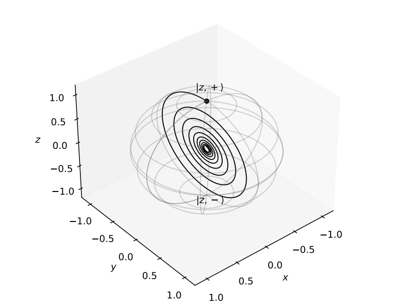
- **Approximation:**
  - Weak coupling approximation
  - Born-Markov approximation
  - Secular time approximation
  - Ensemble average evoluion

$$
\begin{aligned}
\frac{\delta\tilde{\rho}_S}{\delta t}(t) = & -i[\tilde{H}_D(t), \tilde{\rho}_S(t)]_{\text{sec}} \\
& - \int_0^\infty d\tau [\tilde{H}_D(t), [\tilde{H}_D(t-\tau), \tilde{\rho}_S(t)]]_{\text{sec}}\, e^{-\tau/\tau_c} \\
& - \int_0^\infty d\tau \, \mathrm{Tr}_R[\tilde{H}_{SR}(t), [\tilde{H}_{SR}(t-\tau), \tilde{\rho}_S(t) \otimes \tilde{\rho}_R^{eq}]]_{\text{sec}}\, e^{-\tau/\tau_c}
\end{aligned}
$$

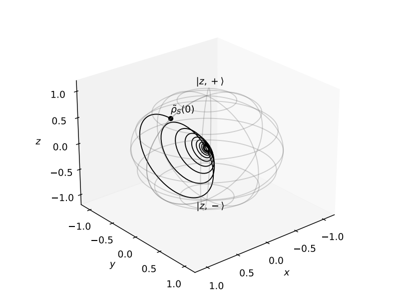
Driven Dissipative Spin 1/2 system

- Hamiltonians:
  - $H_S^\circ = \omega_0 I_z$
  - $H_D = 2\omega_1 \cos(\omega_0 t) I_x$
  - $H_{st}(t) = \sum_j f_j(t) \, |\phi_j\rangle\langle\phi_j|_R$,
    $f_j(t) \rightarrow$ independent Gaussian, δ-correlated white noise
    $\{|\phi_j\rangle\} \rightarrow$ eigenbasis of H_R°
  - $H_{SR} = \omega_{SR}(I_+ R_- + I_- R_+)$
- Start with state $\tilde{\rho}_S(0) = |\psi_0\rangle\langle\psi_0|$
- Evolve by QME

Driven Dissipative Spin 1/2 system



(a) $|\psi_0\rangle = |z,+\rangle$



(b) $|\psi_0\rangle = \frac{3}{5}|x,+\rangle + \frac{4}{5}|z,+\rangle$

- Going inside the Bloch sphere (mixed state).
- After a long time $\tilde{\rho}_S \to \frac{1}{2}\mathbb{1}$ (maximally mixed state)
- Non-unitary evolution.

What’s Next

What's Next

- quantum speed limit
- quantum optimal control

THANKS TO ALL

QUESTIONS?