Composite fermion approach to FQHE

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Outline

- IQHE
- FQHE: Laughlin's approach
- FQHE: Composite fermion

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Little bit history

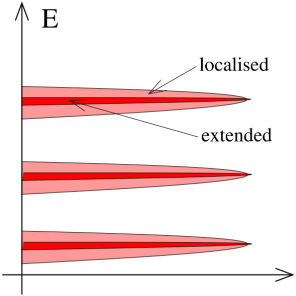
- 1879 - *Classical Hall effect* (E. Hall)
- 1980 - *Integer QHE* (K. von Klitzing)
- 1982 - *Fractional QHE* (Tsui, Störmer, Laughlin)
- 1983–88 - *Hierarchical models* (Haldane, Halperin)
- 1989 - *Chern-Simmons theory* (E. Witten)
- 1989 - *Composite fermion* (J. Jain)

IQHE: Quantized resistivity and Robustness

- Observation: 2DEG with $\vec{B} = B\hat{z}$ and $\vec{E} = E\hat{x}$. The resistivity tensor is

$$\rho_{xx} = 0, \quad \rho_{xy} = \frac{2\pi\hbar}{e^2\nu}$$

- **Robustness:**
  - Disorder broadens the LL.
  - Disorder $\longrightarrow$ localized bulk state
  - Confining potential $\longrightarrow$ chiral edge states (extended)



IQHE: Symmetric gauge and Corbino ring geometry

- Choose Symmetric gauge: $\vec{A} = \frac{1}{2}(-yB, xB)$
- Eigenstates:

$$\psi_{LLL,m} = \left(\frac{z}{l_B}\right)^m e^{-|z|^2/4l_B^2}, \quad z = x - \imath y$$

  - looks like ring. Radius of mth ring, $r = \sqrt{2m}l_B$
- $\hat{L}\,\psi_{LLL,m} = m\hbar\,\psi_{LLL,m}$.
  $m \longrightarrow$ angular momentum (degeneracy of LLL).
- Many-particle Eigenstates: [*Vandermonde determinant*]

$$\Psi_{LLL} = \begin{vmatrix} 1 & 1 & \cdots & 1 \\ z_1 & z_2 & \cdots & z_N \\ \vdots & \vdots & & \vdots \\ z_1^{N-1} & z_2^{N-1} & \cdots & z_N^{N-1} \end{vmatrix} e^{-\sum |z_i|^2/4l_B^2} = \prod_{i<j}(z_j - z_i)e^{-\sum |z_i|^2/4l_B^2}$$

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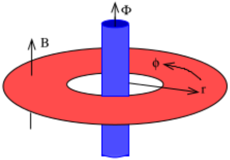
Corbino disc geometry:

$$t = 0 \longrightarrow t = T$$

$$\Phi = 0 \longrightarrow \Phi = \Phi_0 = \frac{2\pi\hbar}{e}$$

- Generate transverse $\vec{E}$ via time-varying Φ (Faraday's law)
- Results radial current flow. $\sigma_{xy} = \frac{e}{\Phi_0}$.



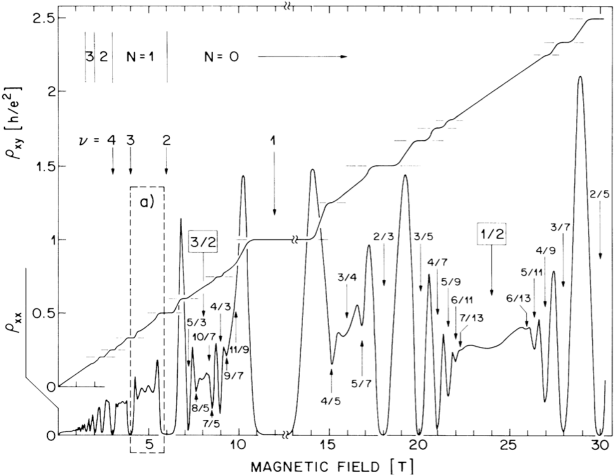
Spectral flow:

$$\Phi = 0 \longrightarrow \Phi = \Phi_0$$

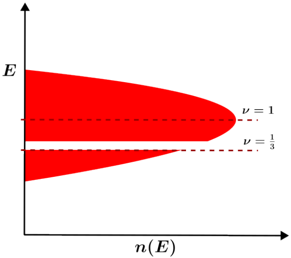
$$\psi_{LLL,m}(\Phi = 0) \longrightarrow \psi_{LLL,m}(\Phi = \Phi_0) = \psi_{LLL,m+1}(\Phi = 0)$$

FQHE: Observations

- plateaus when ν is fraction.
- Almost all the denominator of ν is odd.



FQHE: Why interaction?



- Mixing of Landau levels even without disorder $\longrightarrow$ Coulomb interaction.
- Localization appears to be valid only for strong disorder. Several experiments observe deviation for weak disordered system.
- Expt. shows for high quality samples, there is no direct transition between two integrally quantized plateaus in the lowest two Landau levels.

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FQHE: Laughlin's Formalism

- Full Hamiltonian:

$$H = \sum_j \frac{1}{2m_b} \left[\frac{\hbar}{\iota} \nabla_j + \frac{e}{c} \mathbf{A}(\mathbf{r}_j) \right]^2 + \frac{e^2}{\epsilon} \sum_{j<k} \frac{1}{|\mathbf{r}_j - \mathbf{r}_k|} + \sum_j U_{\text{dis}}(\mathbf{r}_j) + g\mu_B \boldsymbol{B} \cdot \boldsymbol{S}$$

- **Goal :** Wave-function for $\nu = 1/m$ plateaus with *no disorder* and *fully polarized* electron *confined in LLL*.

$$H = \mathcal{P}_{LLL} \frac{e^2}{\epsilon} \sum_{j<k} \frac{1}{|\boldsymbol{r}_j - \boldsymbol{r}_k|} \mathcal{P}_{LLL}$$

- **! Problem :** Cannot treat the Coulomb Interaction as perturbation.

FQHE: Laughlin's Formalism

- **Laughlin's approach:**
  1. Ansatz: $\psi_m(\{z_i\}) = F(\{z_i\})e^{-\sum |z_i|^2/4l_B^2}$. $F(\{z_i\})$ is antisymmetric.
  2. Pairwise function: $F(\{z_i\}) = \prod_{i<j} f(z_i - z_j)$
  3. Requirement 1: $f(-z) = -f(z)$
  4. Requirement 2: eigenstates of Total AM,

$$z \mapsto z e^{\iota\theta}, \quad f(z) \mapsto e^{\iota m\theta} f(z)$$

- Laughlin states:

$$\psi_m(\{z_i\}) = \prod_{i<j} (z_i - z_j)^m e^{-\sum_{i=1}^{N} |z_i|^2/4l_B^2}, \quad m \in \text{odd } \mathbb{N}$$

- Degeneracy of LLL = mN.
- Filling factor $\nu = \frac{1}{m}$.
- **Where are the other filling factors?**

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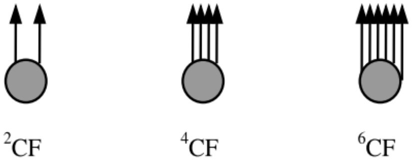
CF: What is Composite Fermion ?

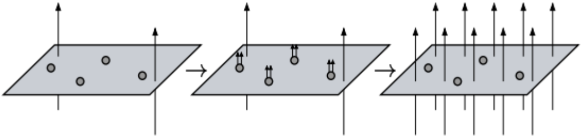
- A *composite fermion (CF)* is an electron bound to even number of vortices or flux quanta.



- **Why is 'even number'?** To preserve Fermi statistics.

CF: Mean Field approximation

- **Goal:** Strongly interacting fermion problem $\leftrightarrow$ weakly interacting CF problem.



- Consider non-interacting electrons at $\nu^* = \rho\Phi_0/|B^*|$.
- Attach $2p$ flux quanta. $p \geq 1$.
- Spread flux quanta adiabatically. [*Mean Field*]
- Eff. magnetic field: $B = B^* + 2p\rho\Phi_0 \geq 0$
- New filling factors $\nu = \rho\Phi_0/|B|$ gives

$$\nu = \frac{\nu^*}{2p\nu^* \pm 1}$$

CF: Another way

- Consider ^{2p}CFs of density $n = \nu B/\Phi_0$ in 2D.
- Berry phase due to closed loop taken by a CF around area A:

$$\gamma = 2\pi\left(\frac{AB}{\Phi_0} - 2pnA\right)$$

- Magnetic field experienced by CF is B^* such that

$$\gamma = 2\pi\frac{AB^*}{\Phi_0}, \quad B^* = B - 2pn\Phi_0$$

- CF filling factor: $n = \nu^*|B^*|/\Phi_0 = \nu B/\phi$

$$\nu = \frac{\nu^*}{2p\nu^* \pm 1}$$

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CF: Filling factors of FQHE

- New filling factors:

ν^*	Filling Fractions ($\nu = \frac{\nu^*}{2p\nu^* \pm 1}$)	
1	$1, \frac{1}{3}, \frac{1}{5} \dots$	(Laughlin state)
2	$\frac{2}{5}, \frac{2}{9} \dots$	
$\vdots$	$\vdots$	

- For $p = 1$ (i.e. ^{2}CF flavor) case
  - For $B^* > 0$ (along the flux):

$$\nu_> = \frac{\nu^*}{2\nu^* + 1} = \frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{5}{11}, \frac{6}{13}, \dots$$

  - For $B^* < 0$ (opposite to the flux):

$$\nu_< = \frac{\nu^*}{2\nu^* - 1} = \frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \frac{5}{9}, \frac{7}{13}, \frac{6}{11} \dots$$

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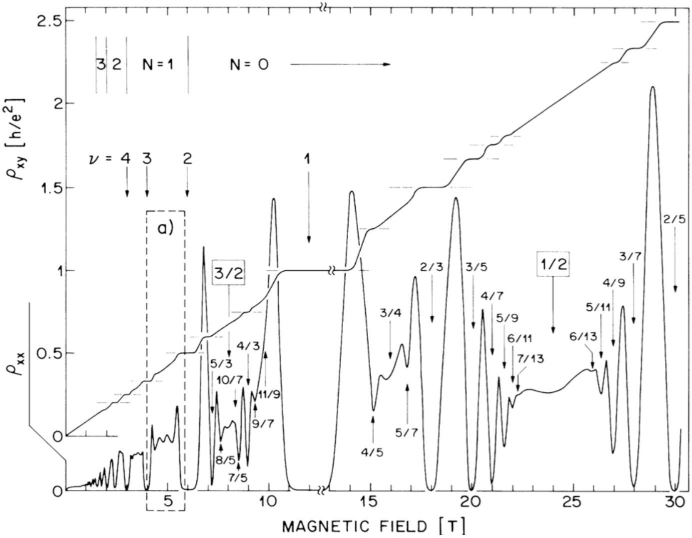
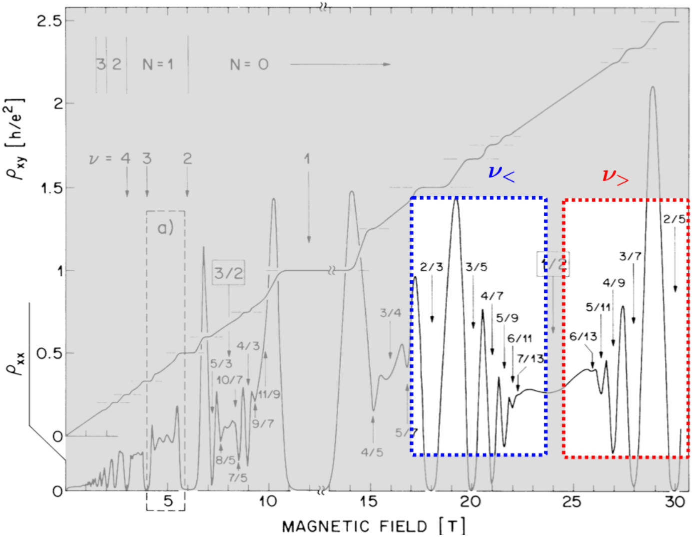
CF: Filling factors of FQHE

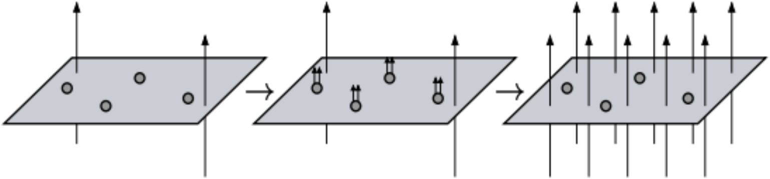


CF: Filling factors of FQHE



CF: IQHE and FQHE connection

IQHE	CF	FQHE
Non interacting Electron	Composite fermion	Interacting electron
B^*	$B = B^* + 2pn\Phi_0$	B
$\nu^* \in \mathbb{N}$	$\nu^* \in \mathbb{N}$	$\nu = \frac{\nu^*}{2p\nu^*+1} \in \mathbb{Q}_{>0}$



- The wave-function with a quasi-hole at position η:

$$\psi_m^{qh}(\{z_i\},\eta) = \prod_{i=1}^{N}(z_i-\eta)\prod_{i<j}(z_i-z_j)^m e^{-\sum_{i=1}^{N}|z_i|^2/4l_B^2}$$

- **Fractionalization:**
  - Berry phase due to closed loop taken by quasi-hole around flux Φ:

$$\gamma = \frac{e}{m}\frac{\Phi}{h}.$$

    Fractional charge, $e^* = +\frac{e}{m}$.
  - *Fractional statistics*: Exchanging two such quasi-holes gives an phase of $\exp\left(\imath 2\pi\frac{1}{m}\right) \longrightarrow$ **Anyons**.

CF: Connection with Laughlin's formalism

- Return to Spectral flow of corbino disc: Flux at origin.

$$\Phi = 0 \longrightarrow \Phi = \Phi_0$$
$$\psi_m \longrightarrow \psi_{m+1} = \left(\prod_{i=1}^{N} z_i \right) \psi_m$$

- Compare with quasi-hole wave function:

$$\psi_m^{qh}(\{z_i\}, \eta) = \prod_{i=1}^{N} (z_i - \eta) \prod_{i<j} (z_i - z_j)^m e^{-\sum_{i=1}^{N} |z_i|^2 / 4 l_B^2}$$

- For interacting system, its a quasi-hole state with hole located at $\eta = 0$.
- Hall conductivity $\sigma_{xy} = \frac{e^*}{\Phi_0} = \frac{e}{\Phi_0} \frac{1}{m}$ [*charge fractionalization*]
- **Flux at** $\eta = 0 \longleftrightarrow$ **Quasi-hole at** $\eta = 0$

CF: Connection with Laughlin's formalism

- From Laughlin wave-function:

$$\psi_m(\{z_i\}) = \prod_{i<j}(z_i - z_j)^m e^{-\sum_{i=1}^N |z_i|^2/4l_B^2}$$
$$= \prod_{i<j} \underbrace{(z_i - z_j)^{m-1}}_{m-1 \text{ flux at } j} \underbrace{(z_i - z_j)\, e^{-\sum_{i=1}^N |z_i|^2/4l_B^2}}_{\text{many-particle LLL in IQHE}}$$

- Each non-interacting electron sees a flux tube of strength $(m-1)\Phi_0$ on every other electrons.
- $\psi_m(\{z_i\})$ is Mean Field solution to the CF model in LLL.
- **Laughlin** $1/m$ **state** $\longleftrightarrow$ $^{m-1}$**CF flavor of** $\nu^* = 1$.

References

- David Tong, *Lectures on the Quantum Hall Effect*, Lecture Notes, University of Cambridge (2016). Available at `http://www.damtp.cam.ac.uk/user/tong/qhe/qhe.pdf`.
- Jainendra K. Jain, *Composite Fermions*, Cambridge University Press, Cambridge (2007).
- J. K.Jain, “The Composite Fermion: A Quantum Particle and Its Quantum Fluids,” *Physics Today*, **53**(4), 39–45 (2000). doi: `https://doi.org/10.1063/1.883035`.

Thank you all!